# Mathematics Applications and Interpretation 

Internal Assessment

## Golden Ratio: The Secret of Life on our Planet and the Fine-Tuned Position of our Planet

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## Introduction

The existence of life has been baffling scientists for the last few hundred years. Life is unique to our planet in the entire Solar System. But why is that?

I've been interested in the biggest mysteries of our Universe since childhood. Life has been one of them. It is interesting, because it is directly connected with the Golden Ratio and the Fibonacci Sequence, which may be the unique blueprints of our Universe. Found in the conch shells, spiral galaxies, and in the proportions of the human body - this ratio can be a part of the structure of matter. That's why, in this exploration, we shall investigate the special properties of our planet in the Solar System, based on the Golden Ratio, and some other meaningful algebraic values, which can be the reason behind the existence of life on Earth. We will see the harmonic alignment of the Earth in relation to the Sun, moon, and many other interesting properties, like the Habitable Zone in our Solar System, that may be based on the Fibonacci Sequence.

## The Golden Section: Mathematics of Life and Growth

Many researchers found that the Golden Section - connected with the Fibonacci Sequence, Golden Spiral, and the number phi ( $\Phi$ ) - is the blueprint of growth of the living organisms. ${ }^{1}$ That's why, in order to find out why life exists on Earth, we have to learn the basics of the above-mentioned concepts.

## 1. The Fibonacci Sequence

The Fibonacci Sequence, named after a medieval Italian mathematician - Leonardo Fibonacci ${ }^{2}$ - is one of the most common mathematical concepts, found almost everywhere in our Universe. The sequence itself can be expressed as follows:

$$
\begin{gathered}
F_{n}=F_{n-1}+F_{n-2} \\
\text { where } F_{0}=0 \text { and } F_{1}=1
\end{gathered}
$$

It means that every term is the addition of the previous two terms, which goes as follows: $0,1,1,2,3,5,8,13,21,34,55,89,144 \ldots$

## 2. The Number Phi

Connected with the Fibonacci Sequence, is the idea of the irrational number phi $(\Phi)$ that can be calculated from the following formula:

[^0]$$
\varphi=\frac{1+\sqrt{5}}{2}=1.61803 \ldots
$$

Actually, sometimes two mathematical concepts of "phi" are identified:

- The Greater $\Phi$ (spelled "fye"), which is the equivalent of 1.61803...3
- The Lesser $\varphi$ (spelled "fee"), which is equivalent of the Greater Phi subtracted by one $(0.61803 \ldots)^{4}$

Another important thing about phi is that we can obtain it by dividing two consecutive numbers from the Fibonacci Sequence (we have to divide the bigger by the smaller). It is interesting, because the bigger the numbers we divide - the more accurate estimation of phi we get. For instance, taking the $5^{\text {th }}$ and $4^{\text {th }}$ Fibonacci sequence term, we get:

$$
8 \div 5=1.6
$$

Which is not an accurate estimate. But when we take for instance the $20^{\text {th }}$ and $19^{\text {th }}$ term, we get closer and closer to the true value (which is still far apart):

$$
6765 \div 4181=1.618
$$

Therefore, the bigger the numbers from the sequence, the closer the result of their division gets to the true phi. In the table below we can see the first fifteen terms, and the number phi calculated from dividing two of consecutive numbers.

Table 1. The first 15 terms of the Fibonacci Sequence with their ratio, going closer and closer to the original phi, calculated from the two consecutive numbers.
\(\left.$$
\begin{array}{|c|c|c|}\hline \begin{array}{c}\text { Term of the Se- } \\
\text { quence: }\end{array} & \begin{array}{c}\text { Fibonacci Se- } \\
\text { quence }\end{array} & \begin{array}{c}\text { Phi } \\
\text { (calculated from the } \\
\text { two consecutive num- } \\
\text { bers) }\end{array}
$$ <br>

\hline 1 . \& 0 \& Undefined\end{array}\right]\)| 1 |
| :---: |
| 2. |

[^1]| 10. | 34 | 1,617647 |
| :---: | :---: | :---: |
| 11. | 55 | $\mathbf{1 , 6 1 8 1 8 2}$ |
| 12. | 89 | $\mathbf{1 , 6 1 7 9 7 8}$ |
| 13. | 144 | $\mathbf{1 , 6 1 8 0 5 6}$ |
| 14. | 233 | $\mathbf{1 , 6 1 8 0 2 6}$ |
| 15. | 377 | $\mathbf{1 , 6 1 8 0 3 7}$ |

3. The

Golden

## Section and the Golden Spiral

But the most important part of the Fibonacci Sequence is the Golden Section. I think the easiest way to describe the Golden Section, is to read the description of the Greek philosopher Plato (427-347 BC), from his work, The Republic. We can find the number phi of the Golden Section, when a segment is divided unevenly into two, in such a way that "the whole to the longer equals the longer to the shorter," and inversely, "the shorter to the longer equals the longer to the whole. ${ }^{55}$ Algebraically, we can express the Golden Section as follows:

$$
\begin{gathered}
\frac{a+b}{a}=\frac{a}{b}=\varphi \\
\text { where } a>b>0 \text {, and } \\
\text { where } \varphi \text { is the Golden Section }
\end{gathered}
$$



Figure 1. The Golden Section represented visually. Source: https://en.wikipedia.org/wiki/Golden ratio\#/media/File:Golden ratio line.svg (accessed 21.08.2021) $a+b$ is to $a$ as $a$ is to $b$

Connected with the Fibonacci Sequence and the Golden Section, is the idea of the Golden Spiral, based on the terms of the Sequence. We start with drawing a $1 \times 1$ square, then

Figure 2. The Golden Spiral, which can
 be one of the blueprints of life on Earth.

[^2]another one (just like in the Sequence), and then we draw squares of other numbers, just as in the image below. ${ }^{6}$

The Golden Spiral is very interesting, because it may be the blueprint of growth of the living organisms, as it can be found in many creatures - from the simple shells of marine species - to the certain parts of the human body.

## Applications:

Recently, exploring the topic of Golden Ratio, I've came across a very interesting image that shows that the human scale is in the Golden Ratio between the microcosm (that of protons and neutrons), and the macrocosm (that of the entire cosmos). As we can see below, I recalculated the data, and they are a fair approximation of phi. Of course, all of that is based on the approximations, however I found it very interesting, because it shows us the placement of the human lifeform in the Golden proportion with the entire existence.


Figure 3. Human scale is the Golden Section between the macrocosm and the microcosm. (Source: www.secretsinplainsight.com)

Because the powers are in the scientific notation - showing only the orders of magnitude - we can simply divide them to check the results:

$$
26 \div 12=1.625
$$

As we see, it is a very close approximation to the phi value of 1.618 . Therefore, the scale of life can be said to be about in the Golden Section between the entire Cosmos and the microscopic world.

1. The Earth-Moon Ratios

Harmony between the Earth and Moon is incredible. The Moon is so fine-tuned that:

The moon orbits the Earth once every 27.322 days. It also takes approximately 27 days for the moon to rotate once on its axis. ${ }^{7}$

[^3]Because of that, moon looks static for the Earth-observer.
But there is also a connection between those two bodies and the Golden Ratio, as we see from the image ${ }^{8}$ below:


Figure 4. The Golden Section between the Earth and the Moon.
Source: https://www.goldennumber.net/solar-system/ (accessed 28.03.2022)

We can check the above substituting the Earth and Moon radiuses:

$$
\begin{gathered}
\text { radius }_{\text {Earth }}=3960 \text { miles } \\
\text { radius }_{\text {Moon }}=1080 \text { miles } \\
\sqrt{\varphi}=1.27 \\
1.27 x=(1080+3960) \\
1 x=3960 \\
5037=5040 \\
\% \text { error }=\frac{|5040-5037|}{5040}=0.0595 \%
\end{gathered}
$$

## 2. 108: The Secret of Life?

When it comes to the mysteries of our Universe - I have never limited myself only to the mainstream scientific thinking. By being interested in spirituality and numerology - I've came across an interesting article by an Indian Yogi - Sadhguru - who explained the secret of the number 108 in the understanding of life. To quote:

The diameter of the Sun multiplied by 108 equals the distance between Sun and Earth, and the diameter of the Moon multiplied by 108 equals the distance between Earth and Moon. The diameter of the Sun is 108 times the diameter of the Earth. ${ }^{9}$

[^4]I've decided to check if the above assertion is right, and if it can be meaningful for life to operate. The colours above correspond with the calculations below.


Figure 5. The mystical concept of 108 chakras in the human body can be seen in the ratios between the Earth, moon, and the Sun. (Source: https://www.himalayanyogainstitute.com/what-is-so-sacred-about-the-number-108/ (accessed

$$
\begin{gathered}
\text { diameter }_{\text {Sun }}=865000 \text { miles } \\
\text { diameter }_{\text {moon }}=2160 \text { miles } \\
\text { diameter }_{\text {Earth }}=7920 \text { miles } \\
\text { Sun-Earth distance }=92700000 \text { miles } \\
\text { Earth-moon distance }=239000 \text { miles } \\
865000 \times 108=93420000 \\
\text { \%error }=\frac{|92700000-93420000|}{92700000}=0.778 \% \\
2160 \times 108=233000 \\
\text { \%error }=\frac{|239000-233000|}{239000}=0.251 \% \\
865000 \div 108=8010
\end{gathered} \quad \begin{aligned}
& \text { \%error }=\frac{|7920-8010|}{7920}=1.136 \%
\end{aligned}
$$

I've also decided to create some set of formulas, in order to find any other interesting connections. The above quote can be described as the following equations:

$$
\begin{gathered}
S_{d} \times 108=|S E| \\
M_{d} \times 108=|E M| \\
E_{d} \times 108=S_{d}
\end{gathered}
$$

It is also a fact that the diameter of the Moon is about 400 times bigger than the diameter of the Sun. ${ }^{10}$ Because of that fact, and the fact that Moon is placed about 400 times closer to the Earth than the Sun ${ }^{11}$, we can experience the total Solar Eclipse, which is another fine-

[^5]tuned characteristic of the placement of the celestial bodies for life. This can be expressed as follows:
\[

$$
\begin{aligned}
M_{d} \times 400 & =S_{d} \\
|E M| \times 400 & =|S E|
\end{aligned}
$$
\]

It is very interesting, because now we can derive some other interesting formulas, by interchanging specific terms:

$$
\begin{aligned}
&|E M| \times 400=|S E|, S_{d} \times 108=|S E| \\
&|E M| \times 400=S_{d} \times 108 \quad \mid \div 400 \\
& S_{d} \times \frac{108}{400}=|E M|
\end{aligned}
$$

Here we do not only have the correspondence between the Sun's diameter and the EarthMoon distance, but also, the golden ratio as the $\frac{108}{400}=0.27=(\sqrt{\varphi}-1)$. It is further very coincidental, because $(\sqrt{\varphi}-1)$ was the ratio between the Moon's and Earth's radiuses, as can be seen from the previous figure:


$$
\mathrm{M}_{\mathrm{r}}=(\sqrt{\varphi}-1) \times E_{r}
$$

As we can see, the same ratio can be found in the EarthMoon radiuses, and the Sun diameter-Earth-Moon distance.

Another interesting set of formulas can be derived in the following matter:

$$
\begin{gathered}
M_{d} \times 400=S_{d}, S_{d} \times 108=|S E| \\
\left.M_{d} \times 400=\frac{|S E|}{108} \quad \right\rvert\, \times 108 \\
M_{d} \times 400 \times 108=|S E| \\
M_{d} \times 43,200=|S E|
\end{gathered}
$$

Here we see the correspondence between the diameter of the Moon and the SunEarth distance.

I think all of the above-mentioned numerical ratios not be a coincidence, and I included these examples, because of their connection with the Golden Ratio. As can be seen from the Figure 6. below ${ }^{12}$, in the circle, the Golden Ratio can be drawn from a $108^{\circ}$ angle between two of its radiuses.

[^6]

Figure 6. Phi-108 relationship. Source: www.shyamsundergupta.com/amaz-
ing.htm (accessed 29.03.2022)

It may be said that this is a coincidence, since we could use other measurement systems for the angles than the 360 degrees in circle. However, this measurement is also connected with the Sun, Moon, and Earth, as it is a mean of the Solar and Lunar Year, the first consisting of 365.25 days, and the second consisting of 354.36 days. ${ }^{13,14}$ As we see, this ratio is connected with the distances, diameters, and cycles of orbits, which further shows us the fine-tuning of our planet.

I've also decided to find out the radius of the above-shown circle, or the AC and BC distances. I used the sine rule to do that:

$$
\begin{gathered}
\frac{\sin \left(108^{\circ}\right)}{P h i}=\frac{\sin 36^{\circ}}{r} \\
\sin \left(108^{\circ}\right) \times r=\sin 36 \times P h i \\
r=\frac{\sin 36 \times P h i}{\sin \left(108^{\circ}\right)}=\operatorname{approx} 1
\end{gathered}
$$

This result was very interesting, because it shows us not only the further connection with the celestial bodies, but it also shows us the magic of phi and the Fibonacci Sequence, as the closer approximation of Phi we got, the closer the radius will be come to 1. Actually, by substituting 1.618 for Phi, I obtained the result of 0.999979 .

## 3. Golden Ratio and Other Planets

Being now very inspired, I decided to check, if the orbits of planets can correspond with the Golden Ratio, especially with the Fibonacci Sequence. I had heard about the Titius-Bode Law ${ }^{15}$, which explains the distances of the orbits of the planets using a mathematical sequence, which can be described in the following way.

First, we start with a sequence, starting with $\mathrm{F}_{1}=0$ and $\mathrm{F}_{2}=3$, and then every term being multiplied by 2 .

$$
0,3,6,12,24,48,96,192,384 \ldots
$$

Which can be described as:

$$
F_{n}=F_{1}+F_{2} \times 2
$$

[^7]$$
\text { where } F_{1}=0 \text { and } F_{2}=3
$$

Then, we add 4 to this sequence:

$$
4,7,10,16,28,52,100,196,388 \ldots
$$

And then divide it by 10 :

$$
0.4,0.7,1.0,1.6,2.8,5.2,10.0,19.6,38.8 \ldots
$$

As we can see, this sequence is neither geometric nor arithmetic. The formula for it can be described as a combination of two sequences:

$$
a=0.4+0.3 \times k
$$

Where:
a = distance between sun and a particular planet
$k=a$ sequence of the powers of two, starting with $F_{1}=0, F_{2}=1$, and $F_{3}=2$, which goes as follows: $0,1,2,4,8,16,32,64 \ldots$, where depending on the planet, you add a corresponding number (for instance Mercury $=F_{1}$, Venus $=F_{2}$, Earth $=F_{3}$ etc.)

We can check, if the above formula really applies to the planets of the Solar System:

$$
\begin{gathered}
\text { Mercury }=0.4+0.3 \times 0=0.4(1 \text { st term }) \\
\text { Venus }=0.4+0.3 \times 1=0.7(2 n d \text { term }) \\
\text { Earth }=0.4+0.3 \times 2=1.0(3 \text { rd term }) \\
\text { Mars }=0.4+0.3 \times 4=1.6(4 \text { th term }) \\
\text { Asteroid Belt }=0.4+0.3 \times 8=2.8(5 \text { th term }) \\
\text { Jupiter }=0.4+0.3 \times 16=5.2(6 \text { th term }) \\
\text { Saturn }=0.4+0.3 \times 32=10.0(7 \text { th term }) \\
\text { Uranus }=0.4+0.3 \times 64=19.6(8 \text { th term })
\end{gathered}
$$

We can see the application of the Titius-Bode law in the following Table:

Table 2: The Titius-Bode formula applied to the planets of the Solar System, with the Actual Distance in the Astronomical Units with the \%Error.

| Planet | Titius-Bode (AU) | Actual Distance <br> (AU) | \%Error |
| :---: | :---: | :---: | :---: |
| Mercury | 0.4 | 0.39 | $\frac{\|0.4-0.39\|}{0.39}=2.56 \%$ |
| Venus | 0.7 | 0.72 | $\frac{\|0.70-0.72\|}{0.72}=2.77 \%$ |
| Earth | 1 | 1.00 | $0.00 \%$ |
| Mars | 1.6 | 1.52 | $\frac{\|1.6-1.52\|}{1.52}=5.26 \%$ |
| Asteroid Belt | 2.8 | No planet | Cannot resolve |
| Jupiter | 5.2 | 5.20 |  |
| Saturn | 10.0 | 9.54 | $\frac{\|10.0-9.54\|}{9.54}=4.82 \%$ |
| Uranus | 19.6 | 19.19 | $\frac{\|19.6-19.19\|}{19.6}=2.09 \%$ |

Now, l've decided to find out if this law can correspond with the Fibonacci Sequence. To do that, I decided to choose the Mercury as the first term, omitting the first two ' 1 s ' of the sequence, because they are the same, and therefore can be considered as the centre. The results can be shown below:

Table 3: The Titius-Bode formula and its correspondence to the Fibonacci Sequence.

| Planet | Titius-Bode (AU) | Fibo- <br> nacci <br> from $\mathbf{F}_{3}$ | Fibonacci from <br> Mercury = $\mathbf{F}_{3}$ | \% Error |
| :---: | :---: | :---: | :---: | :---: |
| Mercury | 0.4 | 2 | 0.4 | $0.00 \%$ |
| Venus | 0.7 | 3 | $0.4 \times \frac{3}{2}=0.6$ | $\frac{\|0.6-0.7\|}{0.7}=14.3 \%$ |
| Earth | 1 | 5 | $0.4 \times \frac{5}{2}=1.0$ | $0.00 \%$ |
| Mars | 1.6 | 8 | $0.4 \times \frac{8}{2}=1.6$ | $0.00 \%$ |
|  |  |  |  |  |


| Asteroid Belt | 2.8 | 13 | $0.4 \times \frac{3}{2}=2.6$ | $\frac{\|2.6-2.8\|}{2.6}=7.69 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| Jupiter | 5.2 | 21 | $0.4 \times \frac{21}{2}=4.2$ | $\frac{\|4.2-5.2\|}{4.2}=23.8 \%$ |
| Saturn | 10.0 | 34 | $0.4 \times \frac{34}{2}=6.8$ | $\frac{\|6.8-10.0\|}{6.8}=47.1 \%$ |
| Uranus | 19.6 | 55 | $0.4 \times \frac{55}{2}=11.0$ | $\frac{\|11.0-19.6\|}{11.0}=78.2 \%$ |

As we see, planets that are the most suitable for the existence of life are in an almost exact Golden Ratio. Here we talk about the Goldilocks Zone ${ }^{16}$, which term refers to the zone that can be hospitable for life, and which is situated between Venus and the Asteroid Belt, with Earth and Mars the most suitable. ${ }^{17}$ So who knows, if our Solar System does not look like a giant Fibonacci Spiral, which is the blueprint for the motions and orbits of planets?

## Reflection

As we can see, this investigation shows us plenty of interesting mathematical correlations between the placement of our planet in the Solar System and the Golden Ratio. These can be the key components for the creation and sustainability of life on Earth, as some researchers and scholars suggested the Golden Ratio to be. ${ }^{18}$ It is interesting, because our Solar System can be also based on the Fibonacci Sequence, so who knows if it isn't one big Golden Spiral, which starts to be distorted with the increasing distance. It is also very fascinating that the ratios between the Earth, Moon, and Sun diameters, and their distances, correspond with each other so well.

For the future studies, it would be nice to analyse the movement of the celestial bodies, and their correspondence to the Golden Spiral. I would also be interested in searching the dependence of the distances and orbits on the frequency and time it takes the body for one rotation around another one.

[^8]
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    ${ }^{4}$ Ibid.

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